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CONTRIBUTION TO THEORETICAL-EXPERIMENTAL INVESTIGATION OF TRANSVERSE VIBRATIONS OF THE STEERING ROD OF A COMMERCIAL MOTOR VEHICLE

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RESEARCH ARTICLE

ABSTRACT: During exploitation, commercial motor vehicles are exposed to vibrational loads that lead to fatigue of the users and materials of their components. Therefore, they need to be studied in the earliest design phase, using mathematical models, experiments, or their combinations. In theoretical considerations, vibrations of concentrated masses are usually observed, although recently, with the development of numerical methods (especially finite element methods), attention is also paid to the vibrations of elastic systems of these vehicles. In such cases, simplifications are usually made, especially regarding exploitation conditions and the interrelationships of vehicle components. This paper attempts to develop a method for identifying vibrational loads on the steering rod under exploitation conditions, using a two-parameter frequency analysis with the use of 2D Fourier transform. The applicability of the procedure is illustrated on an idealized model of the steering rod, and the conducted research has shown that the two-parameter frequency analysis can also be used to generate transverse vibrations of the steering rod in laboratory conditions.

KEY WORDS: *Commercial motor vehicle, steering rod, transverse vibrations, two-parameter frequency analysis*

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PRILOG TEORIJSKO-EKSPERIMENTALNOM ISTRAŽIVANJU POPREČNIH VIBRACIJA SPONE UPRAVLJAČA TERETNOG MOTORNOG VOZILA

REZIME: Teretna motorna vozila su, tokom eksploatacije, izložena vibracijskim opterećenjima koja dovode do zamora korisnika i materijala njihovih agregata. Zbog toga se one moraju izučavati još u najranijoj fazi projektovanja, uz korišćenje matematičkih modela, eksperimenata, ili njihovih kombinacija. U teorisjkim razmatranjima se, obično, posmatraju vibracije koncentrisanih masa, mada se, u poslednje vreme, sa razvojem numeričkih metoda (posebno metode konačnih elemenata), pažnja poklanja i vibracijama elastičnih sistema pomenutih vozila. Tada se, obično, čine idealizacije, naročito u pogledu eksploatacionih uslova i međusobnih veza agregata vozila. U ovom radu je učinjen pokušaj razvoja metode za identifikaciju vibracijskih opterećenja spone upravljača u eksploatacionim uslovima, pri čemu je za dvo-parametarsku frekventnu analizu korišćena 2D Furijeova transformacija. Ilustracija mogućnosti primene postupka je izvršena na idealizovanom modelu spone upravljača, a izvršena istraživanja su pokazala da se dvo-parametarska frekventna analiza može koristiti i pri generisanju poprečnih vibracija spone upravljača u laboratorijskim uslovima.

KLJUČNE REČI: *Teretno motorno vozilo, spona upravljača, poprečne vibracije, dvo-parametarska frekventna analiza*

CONTRIBUTION TO THEORETICAL-EXPERIMENTAL INVESTIGATION OF TRANSVERSE VIBRATIONS OF THE STEERING ROD OF A COMMERCIAL MOTOR VEHICLE

Miroslav Demić

INTRODUCTION

During exploitation, commercial motor vehicles are exposed to vibrational loads that lead to fatigue of the users and materials of their components [1,2]. Therefore, they need to be studied in the earliest design phase, using mathematical models, experiments, or their combinations.

In theoretical considerations, vibrations of concentrated masses are usually observed, although recently, with the development of numerical methods (especially finite element methods [3-5]), attention is also paid to the vibrations of elastic systems of vehicles. In such cases, simplifications are usually made, especially regarding exploitation conditions and interrelationships of vehicle components [6]...

The specificity of exploitation conditions for commercial motor vehicles is their random nature [6], which significantly complicates theoretical considerations using models, making experiments practically irreplaceable. Despite significant progress in the development of software for automatic vehicle design and calculation [5], the final judgment on their characteristics is based on experimental research. Therefore, experimental methods remain important today.

When it comes to the steering system of a commercial motor vehicle subject to random excitations that cause random vibrations, the problem of identifying their parameters often arises. In this regard, methods for their identification have been developed, such as modal analysis [7,8]. In laboratory conditions, vibration modes are practically determined. However, a problem arises when real exploitation conditions are necessary to generate transverse vibrations of the steering rod on test benches because the modal analysis does not provide sufficient possibilities for generating signals in the time domain...

As known [9], during the calculation of the steering rod, its distortion verification is often performed. Since there is an axial force in the steering rod of commercial motor vehicles, there is a risk of distortion, so it was considered useful to further study its transverse vibrations in this paper.

In this sense, this paper aims to develop procedures for identifying parameters of transverse vibrations of the steering rod, which will enable their generation in laboratory conditions. One possibility is frequency analysis using Fourier transformation, which allows determining the frequency content of a signal by calculating the spectrum magnitudes and phase angle [10]. By using inverse Fourier transformation, the original time-dependent signal can be generated, which is routinely performed in cases when the signal depends only on time [10].

However, vibrations of elastic systems depend on multiple parameters (dimensions and time) [11-14], which suggests that a multi-parameter Fourier transform should be used. In the case of an idealized model of the steering rod, which is modeled as an elastic beam, transverse vibrations change along its length and depend on time, so-called two-parameter Fourier transformation (2D) must be applied [14]

This paper will analyze the possibilities of applying the two-parameter Fourier transform to create conditions for investigating transverse vibrations of the steering rod in laboratory conditions.

Therefore, the general expression for the Fourier transformation in the case of multiple variables [14] is given:

$$F(\xi_1, \xi_2, \ldots, \xi_n) = \int_{R^n} e - 2\pi i \left(x_1\zeta_1 + x_2\zeta_2 + \ldots + x_n\zeta_2 \right) * \\ f\left(x_1, x_2, \ldots, x_n\right) dx_1 dx_2 *** dx_n \quad , \tag{1}$$

where:

- $f(x_1, x_2 \ldots. x_n)$ - a function of n variables,
- $x_1, x_2, \ldots. x_n$ - variables,
- $\xi_1, \xi_2 \ldots\ldots \xi_n$ - circular frequency, and
- $\int_{R^n}$ - multiple integral (for 2D - double, 3D - triple, etc.)

1. METHOD

As already mentioned, this study aims to explore the possibility of applying two-parameter frequency analysis (2D Fourier transform) in identifying the parameters of transverse vibrations of the steering rod of commercial vehicles. In the absence of experimental data on registered transverse vibrations of the rod, the method is illustrated with data obtained from dynamic simulation using its mathematical model. It was considered appropriate to first define in detail the load that the steering rod is subjected during exploitation.

First, the kinematics of the steering system will be defined, Figure 1.

$$AB = l_p \varphi = l_s \beta \\ \varphi = \frac{\gamma(t)}{i_u} \quad , \\ \beta = \frac{l_p \gamma(t)}{l_s i_u} \tag{2}$$

where:

- γ(t) - steering wheel angle,
- l_p - length of the steering arm, and
- l_s - length of the steering rod.

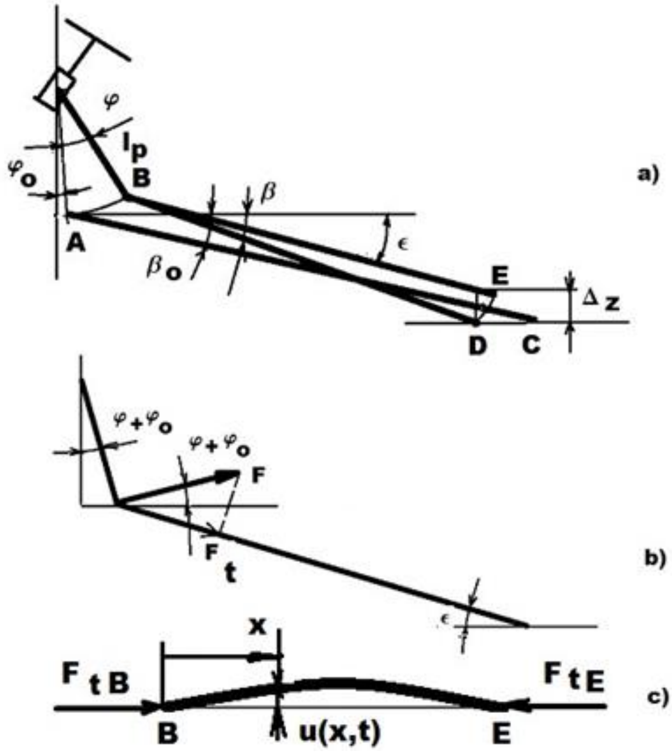

Figure 1. Schematic representation of the kinematics of the steering rod a), force b), and rod transversal vibration model c)

Due to the relative displacement of the joint of spindle arm D, a change of the angle in the vertical plane occurs depending on the sign of (it decreases for positive values and increases for negative values), i.e.:

$$CD = l_s\beta_1 \approx l_s \sin\beta_1 \approx \Delta z$$

$$\beta_1 = \frac{\Delta z}{l_s} \qquad , \tag{3}$$

Finally, based on (2) and (3), the angle of the steering rod in the vertical plane can be calculated (assuming that the steering rod joint moves upwards):

$$\varepsilon = \beta_0 + \frac{l_p\gamma_p(t)}{l_s i_u} - \frac{\Delta z}{l_s}, \tag{4}$$

where:

- β_0 - is the angle of the steering rod in the static position of the vehicle.

From Figure 1 b), it can be seen that the axial force of the steering rod represents the projection of the force *F* to the its direction, i.e.:

$$F_t = F\cos(\varphi_0 + \varphi + \varepsilon), \tag{5}$$

The force F can be expressed in terms of the torque at the steering wheel M, the gear ratio in the steering system i_u, and the steering efficiency η_u :

$$F = \frac{M(t) i_u \eta_u}{l_p}, \tag{6}$$

For further analysis, a simplified model of the steering rod shown in Figure 1 c) is used. When defining a model to describe the transverse vibrations of the steering rod, the following assumptions were made:

- the curvature of the rod is small during transverse vibrations,
- the influence of torsional, and other dynamic loads is neglected,
- the cross-section of the rod is tubular with constant radii along its length,
- the mass of the rod is uniformly distributed along its length,
- the influence of friction and clearances in the rod joints is neglected, and
- the steering rod moves in the vertical plane.

Considering that the partial differential equation that describes the transverse vibrations of an elastic beam, as detailed in [11-13], will not be done here, only the final form will be presented. Based on the introduced assumptions, the forced transverse vibrations of the rod are described by the partial differential equation [11]:

$$\frac{\partial^2 u}{\partial t^2} + \frac{EI_x}{\rho A} \frac{\partial^4 u}{\partial x^4} = F(x,t), \tag{7}$$

where:

- $u = u(x, t)$ - transverse vibrations of the rod,
- x- coordinate along the length of the rod,

F(x, t) - perturbing axial force (excitation function) originating from the driver's action on the steering wheel and the random nature of micro-unevenness of the road,

- t - time,
- I_x - moment of inertia of the cross-section of the rod,
- ρ - material density of the rod,
- E - modulus of elasticity, and
- A - cross-sectional area of the rod.

The area and moment of inertia of the tubular cross-section of the steering linkage are given by the expression:

$$A = \pi\left(R^2 - r^2\right); I_x = \frac{\pi}{4}\left(R^4 - r^4\right), \tag{8}$$

where:

- R - outer radius, and
- r - inner radius of the steering rod.

The magnitudes of forces at the ends of the steering rod were calculated based on experimental data from [15], which were obtained during the free driving of the FAP 1118 vehicle on an asphalt road. The corresponding spectra of the steering wheel angle, steering wheel torque, and vertical relative displacement of the joint of spindle arm are shown in Figures 2-4.

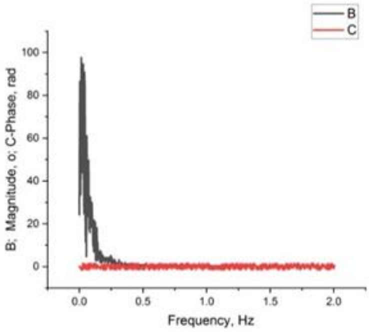

Figure 2. Spectra of the steering wheel angle during free driving on an asphalt road

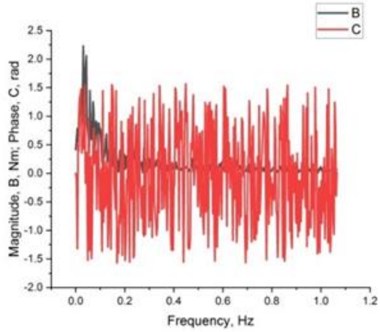

Figure 3. Spectra of the steering wheel torque during free driving on an asphalt road

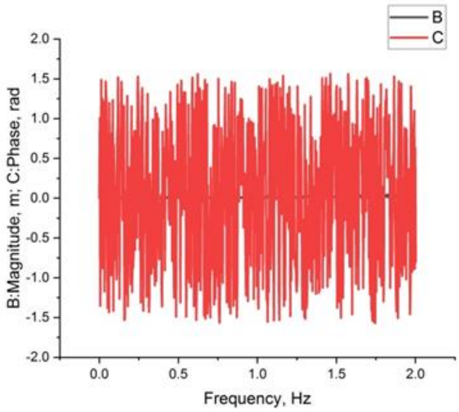

Figure 4. Spectra of the vertical relative displacement of the steering rod joint during free driving on an asphalt road.

The time series of these quantities were calculated using the inverse Fourier transform and are shown in Figure 5. From the displayed figure, it can be seen that the recorded quantities belong to the group of random processes.

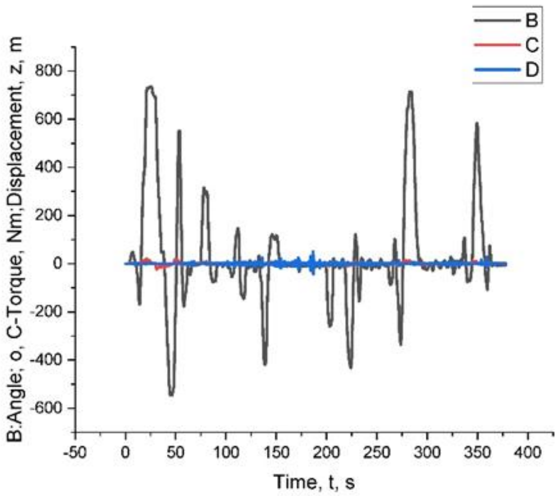

Figure 5. The dependence of the steering wheel angle, steering wheel torque, and relative displacement of the joint of spindle arm, were calculated using the inverse Fourier transformation.

As known [11-14], to find the general integral of the partial differential equation (7), it is necessary to know the boundary and initial conditions. In this specific case, it is assumed that both ends of the steering rod are connected by a spherical bearing [13], so the torques at the ends are equal to zero, and the forces due to vibrations are equal to the applied forces, i.e.:

$$\begin{gathered} -EI_x \frac{\partial^2 u(0,t)}{\partial t^2} = 0; -EI_x \frac{\partial^2 u(l_s,t)}{\partial t^2} = 0 \\ EA\frac{\partial u(0,t)}{\partial x} = F_{tB}; \frac{-EA\partial u(l_s,t)}{\partial x} = F_{tE} \end{gathered}, \tag{9}$$

where:

- F_{tB} and F_{tE} - forces at the ends of the steering rod, which will be discussed later.

For dynamic simulation, it is assumed that the displacements at the spherical joint are zero at the initial moment, i.e.:

$$u(x,0) = 0; u(l_s,0) = 0\,, \tag{10}$$

It should be noted that in the case of a numerical solution of the partial differential equation (7), sometimes it is necessary to introduce additional initial conditions [14].

Based on equation (5), the disturbance force can be defined as:

$$F(x,t) = F_{tB}(0,t) - F_{tE}(l_s,t), \tag{11}$$

The integral of the partial differential equation (7), with boundary and initial conditions (9), (10), and disturbance force (11), can only be sought in the case of harmonic excitation [14]. An attempt was made to solve it using the Wolfram Mathematica 13.2 program [14].

However, difficulties arose with listing numerical data and the fact that the mentioned program solves only partial differential equations up to the second order, so it was decided to solve the problem numerically [16], using the finite difference method. Since this procedure is known from [16], it will not be discussed here, and the problem was solved using a developed program in Pascal.

The dynamic simulation was performed for a steel steering rod using the following data:

E=$2.1*10^5$ N/mm^2; $\rho=8*10^{-6}$ kg/mm^3; R=40 mm; r=30 mm; n_x=40; h_x=40 mm; n_t=1000; h_t=0.02 s; i_u=5; η_u=0.999.

Since the transverse vibrations of the steering rod depend on two parameters, 3D graphics need to be applied for their graphical representation.

The integration of the partial differential equation (7) was performed numerically, and the results are shown in Figure 6. From Figure 6, it can be seen that the transverse vibrations of the steering rod depend on the position along the x coordinate and the time of wave motion observation. Considering the random nature of the excitation forces, the transverse vibrations have a random character, which is by [11].

Based on the aforementioned, for frequency analysis, it is necessary to apply 2D Fourier transform. To implement it, the author developed software in Pascal, for 2D Fourier and Inverse Fourier transform. However, considering the available commercial software on the market, it was deemed appropriate to use Origin 8.5 [17] for further analysis, as potential users will have easier access to this software.

Using the mentioned software, the magnitudes and phase angles of the two-parameter Fourier transform were calculated, and the results, for illustration purposes, are shown in Figures 7 and 8.

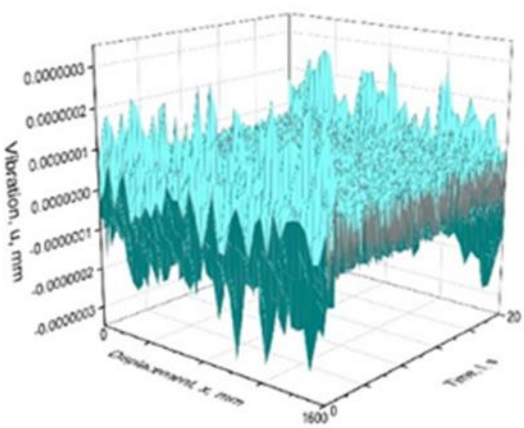

Figure 6. Transverse vibrations of the steering rod

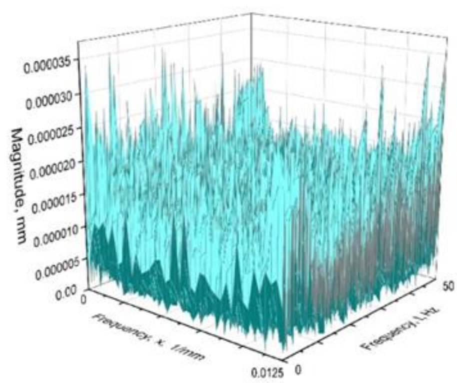

Figure 7. Magnitude spectrum of transverse vibrations of the steering rod

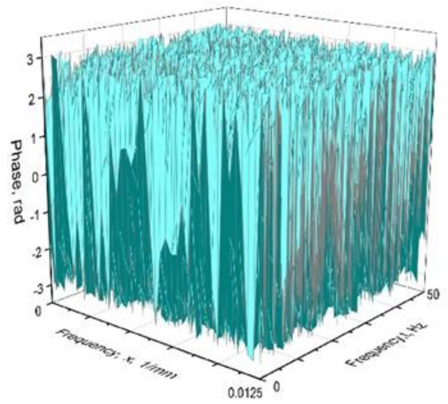

Figure 8. Phase angles of the spectrum of transverse vibrations of the steering rod

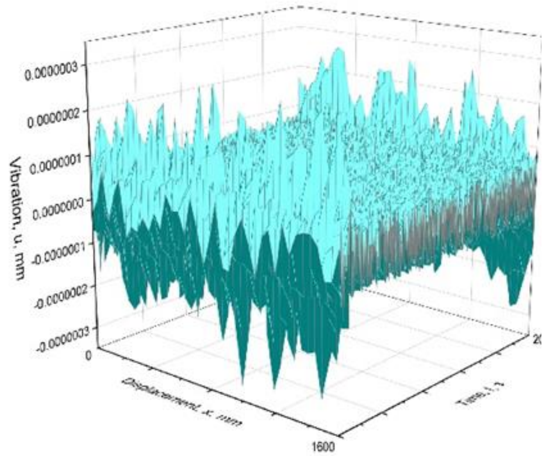

Figure 9. Transverse vibrations of the steering rod obtained by inverse 2D Fourier transformation

2. DATA ANALYSIS

By analyzing the data from Figures 7 and 8, it can be determined that the vibrations (magnitude and phase angle of the spectrum) vary along the length of the steering rod and depend on time. It is evident that the vibrations propagate in the form of random waves, and the magnitudes are higher near the ends of the steering rod, which can be explained by the fact that disturbance forces acted there. This fact confirms the theoretical knowledge about transverse vibrations of the elastic beam [11-14]. The magnitude and frequency of harmonics depend on the design parameters of the steering rod and the time excitation.

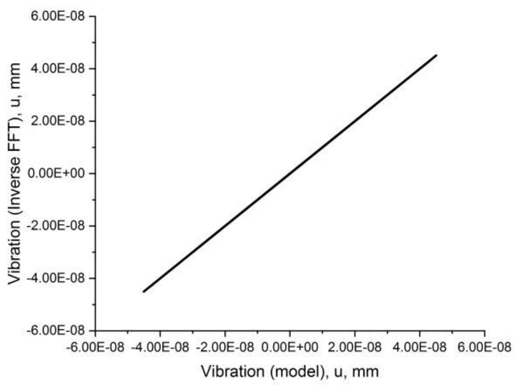

Figure 10. Comparison of transverse vibrations of the steering rod obtained from the model and inverse 2D Fourier transform

It should be noted that the study aims to investigate the possibility of applying a two-variable frequency analysis in testing steering rod in the laboratory. In such cases, it is often required to generate data that correspond to those in exploitation, so it is justified to use 2D Inverse Fourier transform, which allows calculating experimental values of vibrations based on the magnitudes and phase angles of the spectrum. The Inverse Fourier transform can be realized using the mentioned software Origin 8.5 [17]. By applying the mentioned software, the Inverse 2D Fourier transform was calculated, and the results are shown in Figure 9.

To determine the reliability of the data obtained based on the Inverse Fourier transform, the numerical data from Figures 6 and 9 were compared. The result of the comparison is shown in Figure 10. Since their dependence is a straight line, it is evident that the results match, which is also by mathematical laws [14]. Of course, slight differences may occur at the micro level as a result of numerical operations [16]. Considering the high agreement between the data obtained by Inverse 2D Fourier transformation and the experimental data (in this case, calculated based on the mathematical model (7)), the procedure could be used in laboratory conditions [6,18]. During the performance of exploitation tests, it is necessary to record the parameters of transverse vibrations of the steering rod (stresses, displacements, velocities, or accelerations of selected points) along its length, over a longer period. The minimum and maximum frequencies depend on the length of the steering rod, i.e., the length of the time signal and the discretization step.

First, it is necessary to adopt the maximum interesting frequencies fxmax and ftmax, and then the setting of the sensor and the sampling of the time signal are defined based on the expression (Nyquist frequency) [10].

$$h_x = \frac{1}{2 f_{x\max}} \; h_t = \frac{1}{2 f_{t\max}}$$

The minimum interesting frequency is obtained based on the length of the steering rod ($l_s=n_x*h_x$) and the length of the time signal ($t=n_t*h_t$) according to the expressions:

$$f_{x\min} = \frac{1}{n_x h_x} \; f_{t\min} = \frac{1}{n_t h_t}$$

It should be noted that for two-parameter Fourier transformations, there are no explicit procedures for calculating spectral analysis errors, as in the case of 1D Fourier transform [10]. Considering this, as well as the fact that this study aims to illustrate the potential application of two-parameter frequency analysis in investigating transverse vibrations of the steering rod, statistical error analysis was not specifically performed.

Finally, it should be emphasized that the developed procedure has created conditions for analyzing the influence of the integration step on the accuracy and stability of the solution of the partial differential equation (7), the influence of design parameters on transverse vibrations of the steering rod, the influence of disturbance forces, etc. However, considering that the results of dynamic simulation in this study served as a substitute for missing experimental results, it was assessed that a more detailed analysis is not necessary.

3. CONCLUSION

Based on the conducted research, it can be stated that the two-parameter Fourier transform reliably enables the analysis of data on transverse vibrations of the steering rod of commercial motor vehicles.

Calculated magnitudes of spectra and phase angles, using the inverse 2D Fourier transformation, enable the generation of identical vibrations in the laboratory as in exploitation conditions.

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